Upper Bounds for Revenue Maximization in a Satellite Scheduling Problem

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Abstract. This paper presents upper bounds for the problem (SRSS). A compact model of this generalized is defined and enriched with valid inequalities based on task interval reasoning. The non-concavity of the objective function to be maximized is also studied. Finally a Russian Dolls approach combines bounds on nested sub-problems. These first upper bounds for the SRSS problem are compared to best known solutions of the benchmark of the optimization challenge organized by the French OR society.

Keywords: upper bounds, valid inequalities, concave costs, Russian Dolls

AMS classification: 90C90

1 Introduction

An important economical issue for a space agency like the French CNES is the optimization of the schedule of its earth observation satellites. These satellites placed in low orbit around the earth are built around one optical instrument. A mission control center receives observation requests from various customers and is responsible for the control of the satellite. For each revolution, the daily mission management consists in selecting a feasible subset of photograph demands yielding the maximum possible revenue. This selected sequence of images must comply with visibility time-windows, shooting durations and minimum transition times between pairs of photos.
In Agnèse et al. (1995) technical constraints of the considered satellites (the family) restrict time-windows to one single date for each photograph, thus scheduling constraints can be interpreted as mutual exclusions. Since the limited memory available for images storage is taken into account this combinatorial problem can be classified as a generalized 0-1 knapsack problem. Various algorithms have been developed to solve this problem for instance in Bensana et al. (1999) or Vasquez and Hao (2001). Finally Vasquez and Hao (2003) provided tight upper bounds based on 0-1 knapsack linear models.
In the case of agile satellites like those of the family, visibility time-windows are much wider, thanks to the three degrees of freedom of this new generation of satellites. Regardless, in the model proposed by Verfaillie and Lemaître (2001) the storage capacity is assumed to be infinite. The resulting problem called SRSS for

is a
(PC-TSP-TW, Balas 1989) with several additional properties:

1. Two scanning directions are possible for each image acquisition. In other words two shots are possible and mutually exclusive.
2. Some pairs of photos (labeled "stereo") describe two acquisitions of the same geographical area under different angles. Selecting one without the other is forbidden and scanning directions must be identical.
3. Finally, demands are grouped into "polygons" whose revenue is a convex piecewise linear function of the surface covered by selected images. For instance shooting 40% of the surface of a polygon only brings in 10% of its gain.

Many approaches have been proposed to solve the SRSS problem including dynamic programming (Verfaillie and Lemaître 2001), constraint programming (Caseau 2003) and local search (Cordeau and Laporte 2003, Kuipers 2003). However almost no upper bound of the maximal revenue is available. Only Caseau (2003) describes an upper bound based on a preemptive relaxation of the underlying scheduling problem. This bound can be quickly computed at each node of a branch and bound in order to prune the search tree. Yet the gap with best known solutions reaches up to 300% on the largest instances.

The goal of this paper is to propose a linear model of the SRSS problem providing good upper bounds for the maximum revenue. After a brief recall of the problem at stake a compact model will be defined in order to be able to tackle large instances. In section 3 this model is enriched with valid inequalities based on task interval reasoning. The model is expanded through the addition of frontier dates in order to make these cuts more efficient. Then the polyhedral estimation of the convex gain function is improved in section 4. Finally nested sub-problems of growing size are considered with a Russian Dolls Strategy. The obtained upper bounds are compared in section 6 to the best known solutions of the benchmark[1] used for the Roadef Challenge 2003.

2 Linear model

2.1 Problem data and notations

During one revolution around Earth, the observation satellite flies over a certain number of "strips" to be photographed. Each of these photograph demands is characterized by a gain, a visibility time-window and a shooting duration. For each pair of shots, we know the minimum transition time required to maneuver the camera from the end of the first strip to the start of the second strip. The goal is to select and schedule a subset of demands in order to maximize the total revenue.

A problem with strips involves 2 possible acquisitions since for each strip both shooting directions are possible: these shots are numbered 2 -1 and 2 . For each strip , is the index of its stereo twin strip (0 if does not belong to a stereo pair). The duration of shot is and its earliest start, latest start, earliest end and latest end

[1] This benchmark is available at together with a rich definition of the problem and a list of best known results.

dates are respectively noted , , and . The surface of the corresponding strip is . The union of time windows is [$_{min}$, $_{max}$] and we take (0)= (0)= (0)= (0)= $_{min}$ and (2 +1)= (2 +1)= (2 +1)= (2 +1)= $_{max}$. For each pair ≠ ∈[1,2], → is the minimum transition time between and . We assume that → ≥ (otherwise it is obviously under-estimated) and we chose the convention (0→) ( ) $_{min}$ and (→2 1) $_{max}$ ( ). Given { $_1$, $_2$,… $_q$}∈[1,2]^q a sequence of shots we denote by ({ $_1$, $_2$,… $_q$}) the earliest completion time of this sequence or +∞ if the sequence is not feasible.
Let be the number of polygons. The one is characterized by its total surface , its gain when fully selected and the set of shots it contains ⊂ [1,2]. The gain percentage associated with a partial acquisition of a polygon is computed with function :[0,1]→[0,1], piecewise linear and defined by points {(0,0), (0.4,0.1), (0.7,0.4), (1,1)}.

2.2 Basic model

Four variables are associated to each possible acquisition ∈[1,2]:

- $Y_i \in \{0,1\}$ equals 1 if and only if shot is selected.
- $X_{0\to i} \in \{0,1\}$ equals 1 if and only if shot is the first of the selection.
- $X_{i\to 2n+1} \in \{0,1\}$ equals 1 if and only if shot is the last of the selection.
- $T_i \in$[( ), ( )] is the shooting start date of (note that the value of this variable is irrelevant when Y_i=0).

Two continuous variables are associated to each polygon ∈[1,]:

- $S_k \in [0,1]$ is the percentage of surface covered by selected strips.
- $G_k \in [0,1]$ is the corresponding percentage of the polygon gain.

Finally one binary variable is defined for each pair of acquisitions ≠ ∈[1,2].

- $X_{i\to j} \in \{0,1\}$ equals 1 if and only if is shot just after .

Equations constraining these variables are listed below. The acquisition of the same strip in both directions is forbidden by (**1**). Equation (**2**) states stereo constraints: simultaneous selection with identical direction. There is at most one first shot and one last shot (**3**) and each selected acquisition has exactly one predecessor and one successor (**4**). Shooting dates of consecutive shots must respect minimum transition times (**5**). Finally the convex relation between the surface and the gain (equation (**6**)) is modeled with a "special ordered set of type 2" (SOS2 in Xpress-MP (2003)). Section 4 defines valid inequalities strengthening the estimation of G_k in fractional solutions.

$$\forall \in[1,\ ] \quad _{2\ -1} + _{2} \le 1 \qquad \textbf{(1)}$$

$$\forall \in[1,\ ] \quad (\ ) > \quad (_{2\ -1} = _{2\ (\ )-1} \quad _{2} = _{2\ (\ )}) \qquad \textbf{(2)}$$

$$\sum_{\in[1,2\ ]} _{0\to} \le 1 \qquad \sum_{\in[1,2\ ]} _{\to 2\ +1} \le 1 \qquad \textbf{(3)}$$

$$\forall \in[1,2\ ] \quad \sum_{\substack{\in[0,2\ +1] \\ \neq}} _{\to} = \ = \sum_{\substack{\in[0,2\ +1] \\ \neq}} _{\to} \qquad \textbf{(4)}$$

$$\forall \neq \in [1,2\ ]\ - \geq (\ ()+(\rightarrow)) \times\ _{\rightarrow} +(\ ()-\ ())(1-\ _{\rightarrow}) \quad \textbf{(5)}$$

$$\forall \ \in [1,\ ] \quad = \frac{1}{()} \sum_{\in\ ()} () \qquad = (\ ) \quad \textbf{(6)}$$

The objective function to be maximized is ∑ .
An integer solution of this linear program describes a path from image 0 (start) to image 2 +1 (end), with consistent shooting dates. The integrality of and $_{\rightarrow}$ ensures that each acquisition is either inside or outside the selection and that selected images have exactly one predecessor and one successor. Ignoring these requirements, continuous solutions are flows instead of paths, which makes this continuous relaxation similar to the minimum spanning tree relaxation of the Traveling Salesman Problem.

2.3 Dominance rules

This model involves variables and constraints, since there are 2 (2 -1) pairs → . In fact pairs 2 -1→2 are impossible because of constraint (**1**) and pairs → with → are rejected by constraint (**5**). Besides pairs → satisfying **(7)** are dominated. Indeed any solution using this arc would exclude a strip whereas one of the corresponding shots could be inserted between and even if $_i$= ( ) and $_j$= ( ), therefore no optimal solution contains arc → .

$$\exists \ \in [1,\ ]:\ () = 0 \ \wedge$$
$$(\forall \ \in \{2\ -1,2\ \}:\ (\{\ ,\ ,\ \}) = +\infty \wedge \ (\{\ ,\ ,\ \}) = +\infty) \ \wedge \quad \textbf{(7)}$$
$$(\exists \ \in \{2\ -1,2\ \}: \min(\ (),\ () - (\ \rightarrow\ )) - \max(\ (),\ () + (\ \rightarrow\ )) \geq (\ ))$$

This situation is illustrated in **Fig. 1**. Neither nor can be acquired before or after ; and can always be inserted between and .

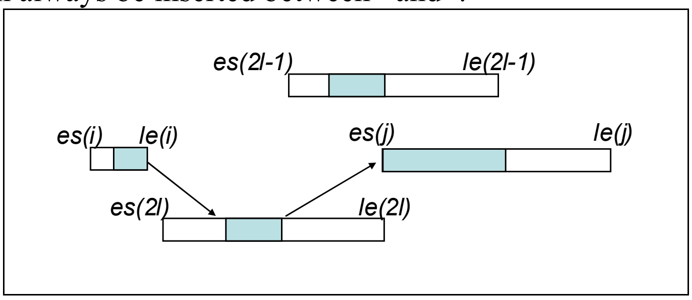

Fig. 1. Dominance rule

2.4 Light model

Removing all impossible or dominated arcs is not sufficient to handle the largest instances (for which 2 ≈1000). Thus a lighter model is introduced in this section.
Considering only the closest neighbors is a classical TSP heuristic. We apply this strategy with the definition of a function η:[1,2]^2→{0,1} such that variable $_{i\rightarrow j}$ will only be created if η =1. Typically we may choose:

$$\eta(\, , \,) = 1 \Leftrightarrow (\exists \; : \, , \; \in \quad) \vee \left(\; (\; \rightarrow \;) \leq (1+\varepsilon) \min_{' \in [1,2 \;]} (\; , \; ') \right) \quad \text{with } \varepsilon>0 \qquad (8)$$

To preserve the validity of computed upper bounds, arcs with η =0 cannot be removed. Two variables $_{\infty\rightarrow}$ and $_{\rightarrow\infty}$ (representing ignored arcs) are added for each shot and equation (**4**) is rewritten as:

$$\forall \; \in [1,2 \;] \quad \sum_{\substack{\in[0,2 \; +1] \\ \eta(\, , \,)=1}} {}_{\rightarrow} + {}_{\infty\rightarrow} = \; = \sum_{\substack{\in[0,2 \; +1] \\ \eta(\, , \,)=1}} {}_{\rightarrow} + {}_{\rightarrow\infty} \qquad (9)$$

The following global equality is also added:

$$\sum_{\in[1,2 \;]} {}_{\infty\rightarrow} = \sum_{\in[1,2 \;]} {}_{\rightarrow\infty} \qquad (10)$$

The last modification of the model is that (**5**) only applies on pairs with η =1

Proposition 1:

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Proof. Given ( ) a solution of the complete model, let ( ) be the solution of the light model defined by :

- $\forall \; \leq 2 \; : \; _i \quad _i \quad _{0\rightarrow i} \quad _{0\rightarrow i} \; , \; _{i\rightarrow 2n+1} \quad _{i\rightarrow 2n+1}$
- $\forall \; \leq \; : \; _k \quad _k \; , \; _k \quad _k$
- $\forall \quad \leq 2 \quad \eta \quad =1 : \; _{i\rightarrow j} \quad _{i\rightarrow j}$
- $\forall \quad \leq 2 \quad \eta \quad =0 :$ if $_{i\rightarrow j}$ =1 then $_{i\rightarrow\infty}$ =1 and $_{\infty\rightarrow j}$ =1

This solution satisfies all constraints of the light model and its cost equals the original one since .■

This property ensures that any upper bound of the optimum of the lighter model is an upper bound of the optimum of the complete model. Therefore this model will be used in all the remaining of this paper. Note that an integer solution of this light model is a set of paths delimited by $\infty\rightarrow$ and $\rightarrow\infty$ arcs. Precedence constraints (**5**) are satisfied inside each path but linking up all these paths is not necessarily possible.

2.5 First valid inequalities

The following remarks hold for each integer solution:

- Cycles $\rightarrow\rightarrow$ are invalid.
- Arcs and shots excluding a shot cannot coexist with this shot.
- Inside any set of images (typically a polygon), the number of selected arcs must remain strictly smaller than the number of selected images (or 0). Besides, as soon as one element of this set is selected, at least one incoming and one outgoing arc must be activated.

The corresponding valid inequalities are worth adding but the resulting model provides poor upper bounds. The average gap with best known solutions on the considered benchmark is around 280% (400% without these first cuts). The key weaknesses of the model are the precedence constraints (**5**) whose linear relaxation is weak due to the possible large term, and the convexity of the gain function which is equivalent to the concavity of a cost function (Erickson et al. 1987). These issues are respectively addressed in sections 3 and 4.

3 Task intervals

During its revolution around earth, the satellite will spend its time acquiring pictures and maneuvering its camera. Therefore the sum of shooting durations ( ) and transition times ($\to$ $\to$) is bounded by the width of the union of time-windows. Concerning transition times represented by variables $_{\infty\to}$ and $_{\to\infty}$ we can introduce the following lower bounds:

$$\forall \in[1,2\ ] \quad (\ \to\infty)=\min_{\substack{\in[1,2\ ]\\ \eta(\ ,\ )=0}} (\ \to\ ) \qquad (\infty\to\ )=\min_{\substack{\in[1,2\ ]\\ \eta(\ ,\ )=0}} (\ \to\ ) \tag{11}$$

However for each pair $\to$ such that $\eta(\ )=0$, two variables $_{\to\infty}$ and $_{\infty\to}$ are defined. Therefore terms $\to\infty$ $_{\to\infty}$ and $\infty\to$ $_{\infty\to}$ may represent the same transition. This leads to the following pair of constraints:

$$\sum_{\in[1,2\ ]} (\ ) + \sum_{\substack{\neq\ \in[0,2\ +1]\\ \eta(\ ,\ )=1}} (\ \to\ )\ _{\to} + \sum_{\in[1,2\ ]} (\ \to\infty)\ _{\to\infty} \le\ _{max} -\ _{min} \tag{12}$$

$$\sum_{\in[1,2\ ]} (\ ) + \sum_{\substack{\neq\ \in[0,2\ +1]\\ \eta(\ ,\ )=1}} (\ \to\ )\ _{\to} + \sum_{\in[1,2\ ]} (\infty\to\ )\ _{\infty\to} \le\ _{max} -\ _{min} \tag{13}$$

3.1 Task intervals inequalities

The above inequalities are enough to decrease the average gap down to 65% but they can also be declined on smaller intervals. This consistency rule ensuring that a set of tasks has a time window wide enough for its total processing time is the core principle of Task Intervals defined in Caseau and Laburthe (1994) for jobshop scheduling. We propose to convert this rule into valid inequalities for our PC-TSP-TW linear model.
For an interval $[\ _{min},\ _{max}]$ the set of photos that must necessarily be acquired within this time window, if selected, is $_{[tmin,tmax]}=\{\ |\ _{min}\le\ \ \le\ \ \le\ _{max}\}$. Restricting arcs to those that are necessarily inside $[\ _{min},\ _{max}]$ would be valid but a finer analysis of time-windows of arcs leads to stronger inequalities. We define $\to$ and $\to$ as the latest start and earliest end of transition $\to$:

$$(\ \to\ )=\begin{cases} (\ ), & =\infty \\ \min(\ (\ ),\ (\ )-(\ \to\ )), & \end{cases} \quad \text{and} \quad (\ \to\ )=\begin{cases} (\ ), & =\infty \\ \max(\ (\ ),\ (\ )+(\ \to\ )), & \end{cases} \tag{14}$$

The latest start of transition period $\to$ is naturally bounded by . The second upper bound is illustrated on **Fig. 2**. When is followed by the movement of the camera must start early enough to be completed before the latest start time of .

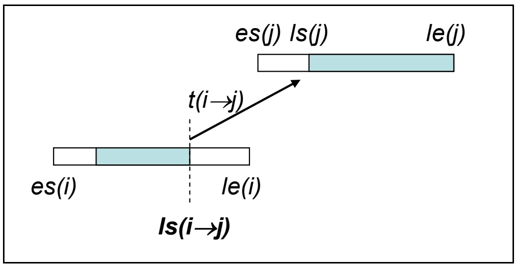

Fig. 2. Latest start of transition $\rightarrow$

These "mandatory" parts of arcs can be used to define the following function, giving for each transition (including $\rightarrow\infty$ and $\infty\rightarrow$) what part of its length will necessarily be included in interval [$_{min}$, $_{max}$].

$$ _{[\ min,\ max]}(\ \rightarrow\ ) = \begin{cases} \rightarrow\ , & \in\ \ \in \\ \min\ \ \rightarrow\ \ _{max}\ \ \rightarrow\ , & \in\ \ \notin \\ \min\ \ \rightarrow\ \ \ \rightarrow\ \ _{min}\ , & \notin\ \ \in \\ 0, & \notin\ \ \notin \end{cases} \qquad (15)$$

When both shots are inside the interval, the full transition necessarily occurs inside the interval. When only $\in$, it can happen that this transition ends outside the interval. In its rightmost position, the portion of arc $\rightarrow$ included in [t_{min},t_{max}] is at most - $\rightarrow$ For instance on **Fig. 2**, if , the part of $\rightarrow$ included in is $\rightarrow$. The case when only $\in$ is symmetrical. Finally arcs with no extremity inside the interval are not included at all. With this function equations (**12**) and (**13**) can be rewritten as follows on a restricted interval [t_{min},t_{max}] :

$$\sum_{\in\ [\ min,\ max\ ]} (\ ) + \sum_{\neq\ \in[0,2\ +1]} {}_{[\ min,\ max\ ]}(\ \rightarrow\ )\ _{\rightarrow} + \sum_{\in\ [\ min,\ max\ ]} {}_{[\ min,\ max\ ]}(\ \rightarrow\infty)\ _{\rightarrow\infty} \leq\ _{max} -\ _{min} \qquad (16)$$

$$\sum_{\in\ [\ min,\ max\ ]} (\ ) + \sum_{\neq\ \in[0,2\ +1]} {}_{[\ min,\ max\ ]}(\ \rightarrow\ )\ _{\rightarrow} + \sum_{\in\ [\ min,\ max\ ]} {}_{[\ min,\ max\ ]}(\infty\rightarrow\ )\ _{\infty\rightarrow} \leq\ _{max} -\ _{min} \qquad (17)$$

As explained in Caseau and Laburthe (1994) only a quadratic number or intervals are relevant. However adding inequalities would be too costly since can take values up to 500. Hence we choose two families of intervals:

- Arbitrary regular partitions of [$_{min}$, $_{max}$] in α intervals (with α from 1 to 5)
- Polygon time windows or unions of pairs of such intervals (when overlapping).

In any case we ensure that [$_{min}$, $_{max}$] is the union of time-windows of included shots. These cuts are very efficient since they overcome the well known weakness of the linear relaxation of precedence inequalities (**5**). In continuous solutions, when a fraction of a shot is selected the same fraction of its shooting duration and transition times are counted in including task intervals. With this family of inequalities, the average gap becomes 44%.

3.2 Additional cuts

Another reasoning can be performed on interval [ ], based on the identification of "exiting" arcs. Such arcs have their origin in and are necessarily the last of the interval when selected. We define a binary function $\gamma^{+}_{[tmin,tmax]}(\ )$, equals to 1 when $\ \rightarrow\ $ is an exiting arc of interval [ ]:

$$\forall \ \in \ _{[\ \min,\ \max]}\ \ \gamma^{+}_{[\ \min,\ \max]}(\ ,\infty)=1 \Leftrightarrow \min_{\substack{\neq \\ \eta(\ ,\ )=0}} (\{\ ,\ \}) > \ _{\max} \tag{18}$$

$$\forall \ \in \ _{[\ \min,\ \max]}\ \forall \ \in [1,2\ \ +1]\ \ \gamma^{+}_{[\ \min,\ \max]}(\ ,\ )=1 \Leftrightarrow \min_{\substack{\neq \\ \in}} (\{\ ,\ ,\ \}) > \ _{\max} \tag{19}$$

Arcs $\rightarrow\infty$ are exiting arcs when no pair with η =0 can be completed before . Indeed it ensures that no such belongs to or precedes an element of . For instance on **Fig. 3**, if $\eta\ =\eta\ =\eta\ =0$, none of these destinations can belong to TI or can be completed before . Concerning arcs $\rightarrow$ with outside , if no shot of can be taken after then is the last shot of . For instance on **Fig. 3**, neither or can be taken after hence when arc $\rightarrow$ is selected, is the last shot of

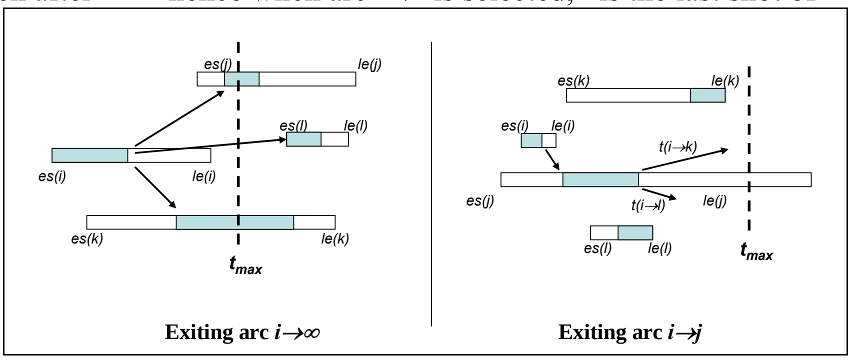

Fig. 3. Exiting arcs

Since can only have one last shot, at most one arc of this kind can be selected for each interval (**20**). Besides, it shall be noted that $\infty\ \ _{\rightarrow\infty}$ can be added to both (**16**) and (**17**) when $\gamma^{+}(\ \infty)=1$, because we can be sure that it cannot represent an incoming arc of the interval.

$$\sum_{\in\ _{[\ \min,\ \max]}} \left(\sum_{\in[1,2\ \ +1]} \gamma^{+}_{[\ \min,\ \max]}(\ ,\ )\ \ _{\rightarrow} + \gamma^{+}_{[\ \min,\ \max]}(\ ,\infty)\ \ _{\rightarrow\infty} \right) \leq 1 \tag{20}$$

In practice inequalities (**20**) (and the symmetric ones based on “entering” arcs) do not significantly improve the obtained upper bounds on the considered benchmark.

3.3 Frontiers

When visibility time windows are wide, the cardinality of is often very small. For instance a photo with $\ _{\min}$ and $\ _{\max}$ would belong only to the global interval (equations (**12**) and (**13**)). To increase the efficiency of task interval inequalities, enriching the model with “frontier dates” proved to be useful. We choose a few frontier dates and each image is split into several mutually exclusive possible shots. For instance if frontiers $\tau_1,\tau_2\ldots\tau_\beta$ intersect time window [ ], is expanded into $2\beta+1$ shots with time windows [$,\tau_1$], [τ_1- $,\tau_1$+], [τ_1,τ_2] …

[τ_β,]. For each frontier at most one crossing shot can be selected. On the other hand shots delimited by frontiers belong to more task intervals. In continuous solutions photos with wide visibility period are partly counted in several task intervals instead of being excluded from all. This extension of the model provides upper bounds 41% above best known solution (in average).

4 Valid inequalities for the convex gain function

The convex relation between the covered surface and the obtained gain for each polygon (function) can make the continuous optimum far above the integral one. The convex hull of pairs ( )$\in[0,1]^2$ satisfying $\leq$ ( ) is the triangle defined by {( )$\in[0,1]^2$ | $\leq$ } that is to say that it contains points like (0.4,0.4) with up to 300% above . In this section we use the definition of $_k$ as the sum of covered surfaces to refine the polyhedral description of

$$\left\{ (\;,\;) \in \{0,1\} \times \Re^{+} \;\middle|\; \leq \left(\frac{1}{(\;)} \sum_{\in (\;)} (\;) \right) \right\} \quad \textbf{(21)}$$

For any bipartition of we use the following notations: = $\cup$, max$_k$'=max( ), max$_k$"=max( ).

Proposition 2:

$$\leq \pi \qquad \pi \qquad \text{with } \pi \; \frac{(\max')}{\max'} \text{ and } \pi = \frac{1-(\max')}{\max''} \quad \textbf{(22)}$$

Proof. Coefficients π and π are the slopes or chords [(0,0),(max'$_k$, (max'$_k$)] and [(max'$_k$, (max'$_k$),(1,1)]. Since is convex we have $\pi \leq \pi$

- If $\leq$ max'$_k$ then $\leq \quad \leq \pi \quad \leq \pi \quad \pi$
- If $>$ max'$_k$ then $\leq \quad \leq \pi$ max'$_k$ π (- max'$_k$) $< \pi \quad \pi$ (-)

All cases are covered hence the inequality is always valid.■

For instance if { } with s$_u$(i)=0.4 and s$_u$(j)=0.6 the point (=1, =0, $_k$=0.4) is cut by inequality $_k\leq$ 0.1 $_1$+0.9 $_2$. It shall be noted that when an upper bound of is available, π can[2] become the slope of [(max'$_k$, (max'$_k$),($_k$, ( ))] what makes (**22**) much stronger For $\geq$3 these inequalities are not facets of (**21**). Its convex hull described in the appendix is made of ! inequalities. In practice we use this convex hull for polygons made of less than 4 photos, and for all polygons we post inequalities corresponding to all bipartitions such that one of both sets has less than 3 elements. It leads to an average gap of 27%. Note that when the 2 inequalities (**22**) are added to the model the SOS2 introduced in section 2.2 can be removed, since each possible selection is correctly estimated by the inequality of the corresponding bipartition. Therefore the only remaining SOS2 are those of polygons of size 5 or more (less than one third of all non singletons polygons). A branch and bound focused on this small number of SOS2 decreases the average gap down to 22%.

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[2] When max'$_k$> we would post Σ $\leq$ | | - 1

5 Russian Dolls Search

The idea of the Russian Dolls algorithm developed by Verfaillie et al. (1996) is to successively solve growing nested sub-problems. Each sub-problem provides a good bound boosting the resolution of the next ones, what makes the whole process much faster than a direct resolution of the whole problem. The goal of this section is to apply this principle to enrich our linear model, in the spirit of Benoist (2002). The considered sub-problems are subsets of polygons. If is an upper bound of the problem restricted[3] to polygons of $\subset [1, \ ]$, then the following inequality is valid:

$$\sum_{\in} \quad \leq \qquad \textbf{(23)}$$

For instance for each singleton of [1,] an exact resolution of the corresponding restricted problem (integer resolution of the complete model 2.2) provides a precious bound of the maximum gain that can be extracted from this polygon. As pointed out in section 4, this cut also provides a finer estimation of the convex gain for this polygon.
This first pass yields an 19% average gap in less than 10 minutes. More precisely, at most 5 minutes are needed to obtain optimal integer solutions for each polygon and another 5 minutes are required to solve the global linear model enriched with all cuts described in previous section.
After this first pass sets of polygons included in intervals similar to those described in 3.1 are considered. The size of these sub-problems allows including more neighbors (adjusting η) and more task interval inequalities. These enrichments combined with cuts (**23**) for included sub-problems and possibly with a truncated branch and bound help obtaining good bounds even when the exact optimum is not available. Finally, nested sub-problems restricted to polygons fully contained in $[\ , \ _{max}]$ are considered, starting with the 5 rightmost polygons and then slowly decreasing from $_{max}$ to $_{min}$ such that the sub-problem contains at most 5 more polygons than the (i-1)th sub problem. We solving this sub-problem, integrality constraints of the five newly added polygons are not relaxed and a branch and bound is performed on this small number of variables.
With this Russian Dolls approach, the final average gap is 12%. However, it shall be noted that this final pass can take several CPU hours.

6 Computational results and conclusion

The following table compares the obtained upper bounds to the best known solutions on the benchmark used for the Roadef Challenge 2003. The gap is computed with formula: (/) 1. The rightmost columns and are gaps obtained at the end of sections 2,3 and 4 respectively.

[3] restricted to D means that $\forall\ k \notin D, \ =0$

Table 1. Final results

					gap			
Test set A	2_9_36	2	10 423 440	10 423 440	**0.0%**	0%	0%	0%
	2_9_66	7	115 710 660	115 710 660	**0.0%**	0%	0%	0%
	2_9_170	25	191 358 231	191 358 231	**0.0%**	0%	0%	0%
	2_13_111	106	563 597 071	601 655 000	**6.8%**	9%	36%	107%
	2_15_170	295	719 417 220	830 248 079	**15.4%**	17%	40%	232%
	2_26_96	483	1 005 301 900	1 188 506 493	**18.2%**	25%	45%	362%
	2_27_22	534	967 910 750	1 199 412 076	**23.9%**	32%	49%	459%
	3_8_155	28	121 680 360	121 680 360	**0.0%**	0%	0%	3%
	4_17_186	147	185 406 780	199 376 544	**7.5%**	10%	34%	66%
	3_25_22	342	425 983 220	510 274 816	**19.8%**	41%	62%	353%
Test set X	2_28_111	428	875 447 480	1 085 340 453	**24.0%**	35%	51%	408%
	2_28_140	522	833 286 610	1 162 118 016	**39.5%**	53%	86%	580%
	2_28_155	375	952 267 030	1 100 321 387	**15.5%**	23%	38%	375%
	2_28_170	446	963 809 499	1 003 712 585	**4.1%**	20%	40%	360%
	2_28_37	408	992 155 179	1 030 371 505	**3.9%**	9%	41%	276%
	2_28_66	379	945 737 319	966 705 352	**2.2%**	6%	18%	265%
	2_28_7	470	977 811 340	1 096 439 092	**12.1%**	19%	39%	316%
	2_28_81	468	878 847 950	1 176 474 568	**33.9%**	45%	78%	432%
	3_28_155	292	462 070 340	474 385 890	**2.7%**	32%	47%	317%
	3_28_96	305	458 107 362	529 529 184	**15.6%**	38%	55%	332%
				Average gaps:	**12.2%**	22%	41%	280%

These results are the best known upper bounds for the SSRS problem. It proves the optimality of known solutions on four instances. Incidentally, on these instances with gap 0% the integer solution of the light model happened to be a solution of a complete problem ($_{i\to\infty}$ = $_{\infty\to i}$=0 ∀). However on other instances, greedy attempts to turn solutions of the light model into real solutions (possibly giving up photos) yielded poor results.

These upper bounds are obtained with a compact linear model enriched with task interval inequalities and cuts dedicated to the convexity of the gain function. A final significant improvement is obtained through a Russian Dolls procedure. The important gap observed on the largest instances shows that the problem remains open: either solutions or upper bounds can be improved.

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Appendix

In this section we study the convex hull of the following set X where is any convex increasing function with (0)=0 and a vector of positive real numbers:

$$= \{(\ , \ ) \in \{0,1\} \times \Re^{+} \mid \ \leq \ (\ ^{T} \ )\} \quad \textbf{(24)}$$

Noting the set of permutations of {1,… }. We will prove that the following polyhedron is the convex hull of X.

$$= \{(\ , \ ) \in [0,1] \times \Re^{+} \mid \ \leq \pi_{\sigma}^{T} \ , \ \forall \sigma \in \ (\ )\} \quad \textbf{(25)}$$

$$\text{where } \pi_{\sigma,} = \left(\sum_{\leq} \ _{\sigma(\ )} \right) - \left(\sum_{\leq -1} \ _{\sigma(\ )} \right) \quad \textbf{(26)}$$

Coefficients π_{σ} are the slopes of chords of . We will note π π.

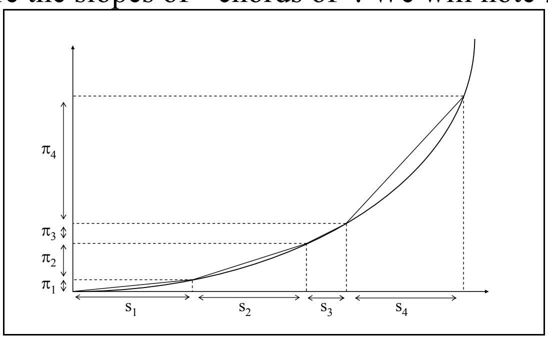

Fig. 4. Coefficients π_i

Proposition 3: $\leq \pi_{\sigma}^{T}$

Proof. For symmetry reasons it is sufficient to prove this for σ=

For (*, *) in X we define ={ ≤ | =1} and we can write:

$$(\ ^{T} \ ^{*}) = \sum_{\in \ ^{*}} \delta \ \text{ with } \delta \ = \left(\sum_{\leq \ , \ \in \ ^{*}} \right) - \left(\sum_{\leq -1, \ \in \ ^{*}} \right) \quad \textbf{(27)}$$

Due to the convexity of we have $\delta_i \leq \pi_i$ thus $\leq \pi^{T}$ is valid.

Let $^{(k)}$ be the vector of $\{0,1\}^{r}$ defined by $_{i}^{(k)}=1 \Leftrightarrow \ \leq$ Equation $= \pi^{T}$ is satisfied for the following +1 points of X: { $\Sigma_{\leq}$ $\in [0, \ ]$}. Since all these points are affinely independent, this inequality defines a facet of X. ■

Proposition 4:

Proof. Let *=(*, *) be a (fractional) point of H. Without loss of generality we can assume that $_i$*≥ $_{i+1}$*. We re-use notation introduced in proposition **3** and we define $^{(k)}$ and π* as:

$$^{(\ )} = \sum_{=1} \pi \ \text{ and } \ \pi^* = \sum_{=1} \pi \quad ^* \qquad \textbf{(28)}$$

Since (*, *) is a point of H it satisfies the valid inequality associated to permutation σ= thus *≤π*. Therefore (*, *) is a convex combination of points of X:

$$\left(\ ^*, \quad ^* \right) = \sum_{=1} (\ ^* - \ ^*_{+1})(\ ^{(\ )}, \frac{^*}{\pi^*} \ ^{(\ )}) + (1 - \ ^*_1)(0,0) \qquad \textbf{(29)}$$

Finally every point of H is a convex combination of points of X. Since X⊆H we conclude that H is the convex hull of X. ■